

Coastal Erosion at Pau Amarelo Beach, Northeast of Brazil

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ABSTRACT

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This article concerns the erosion at Pau Amarelo Beach in Pernambuco, Brazil. The maintenance of this beach is important for tourists and the community and is significant for the local economy. Pau Amarelo is a highly urbanized area with poor coastal plain. The coast is characterized by the presence of lined reefs running along the shore. In order to understand the causes of the erosion at this site, coastline variation, the displacement of urban areas and wave characteristics were analyzed. It is believed that both the human incursions on the backshore and the intensification of longshore currents in some parts of the coast, caused by wave diffraction at the reefs, contribute to the beach erosion. The breakwaters built south of Pau Amarelo are also believed to decrease sediment transport by blocking the drift current. Beach nourishment is suggested as the most suitable solution to the erosion at Pau Amarelo.

ADDITIONAL INDEX WORDS: *Beach erosion, anthropic effects, numerical wave modeling, coastal hydrodynamic, longshore currents.*

INTRODUCTION

Pau Amarelo is an embayed beach in Paulista City, situated 18 km north of Recife, in Pernambuco state, Brazil. The beach is approximately 4 km wide, sheltered by a discontinuous reef system positioned about 300–700 m away from the shoreline (Figure 1).

Close to the beach there are approximately 13,000 inhabitants and the population density reaches 0.9 hab/m², thus making it a densely populated area. Several bars and restaurants, open all year round, can be found at the beachfront (FINEP/UFPE, 2009).

Over the past 60 years, occupation of the first 500 m from the shoreline has increased at Pau Amarelo, changing the natural environment from small communities of fishermen to one of extensive residential and touristic areas. The urbanization has been developed in an unorganized way, with new buildings constructed even on the backshore (Manso *et al.*, 2006).

In general, coastal erosion is a natural process, but human activities can intensify its effects, with erosion occurring mainly in densely populated areas. The causes are related to poor urban planning, which interfere in the natural sediment balance and problems regarding land use on the coast (Lopez and Marcomini, 2013; Souza, 2009).

The erosion process at Pau Amarelo Beach has caused significant economic losses for the local population as well as modifying the natural beauty of the beach (Figure 2a).

The erosion intensified in the 90's and at some locations, residential and commercial properties have been destroyed by the sea. As an emergency solution to a serious erosion event in

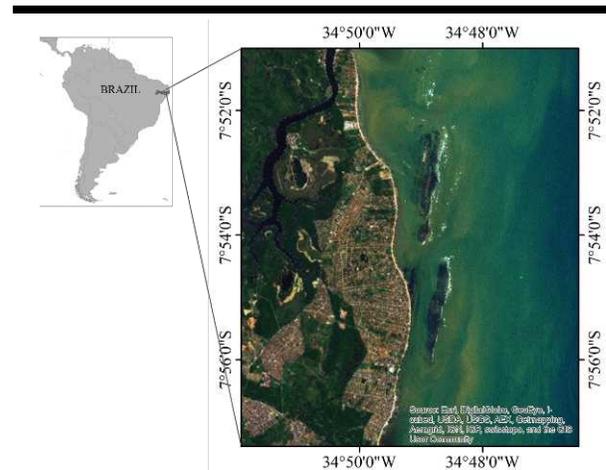


Figure 1. Location of Pau Amarelo Beach.

1999, the city council of Paulista, responsible for the maintenance of the Pau Amarelo Beach, together with private property owners, invested in the construction of a revetment at the beach front, as shown in Figure 2b.

In this context, the aim of the present case study is to identify the natural forces and/or anthropogenic factors causing erosion at Pau Amarelo Beach, through the evaluation of coastline variation, the behavior of waves crossing the reefs and their interaction with the coast.

Geological Aspects

The local coastal zone is composed of Holocene sediments from the Quaternary period (medium and fine sand), which are

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deposited over the crystalline base of the Borborema Province from the Precambrian age. At Pau Amarelo the sediment is siliceous and carbonate, and the mean grain size is 0.34 mm (FINEP/UFPE, 2009).



Figure 2. Destruction of houses, loss of beach, and revetment in front of houses protecting private properties (photos from the authors).

A typical morphological feature along this stretch of the Brazilian coastline are the beachrock ridges running parallel to the coast. They were formed about 6,000 years ago in the Holocene period (Ferreira *et al.*, 2011). They form a unique landscape, enabling the development of reefs, which have a great importance for fishing, coastal protection and tourism (Dominguez *et al.*, 1990).

The coastal waters of northeastern Brazil favor the formation of beachrocks due to the super saturation of calcium carbonate, warm water temperatures and the mesotidal system that generates a drying and wetting cycle of the foreshore. These factors allow the precipitation of calcium carbonate, which acts as a carbonate cement that aggregates the sand and form a sedimentary rock (Ferreira *et al.*, 2011).

Due to the existence of the reef, the local bathymetry exhibits a complex pattern. Because the profile was modified after the consolidation of beachrocks, some parts of the beach face are solid and others not. There is also a channel between the near shore reefs and the beach. As a result, the bathymetry has unusual variations from the beach to the reefs (Figure 3).

Oceanographic Aspects

Pau Amarelo has a tropical Atlantic climate with an average annual temperature of 24° C and annual rainfall of around 2000

mm during winter months, with a drought period for the rest of the year (Melo, 1958).

The wind regime is governed generally by atmospheric pressure distribution; the predominant wind direction in the South Atlantic Ocean is from the southeast (SE). The area of convection of air masses in the tropics is called Hadley cells and surface winds are the Trade Winds, which are considered to be the most constant on the planet. These winds transport heat and humidity from low latitudes to regions of high latitude (Silva, 2003).

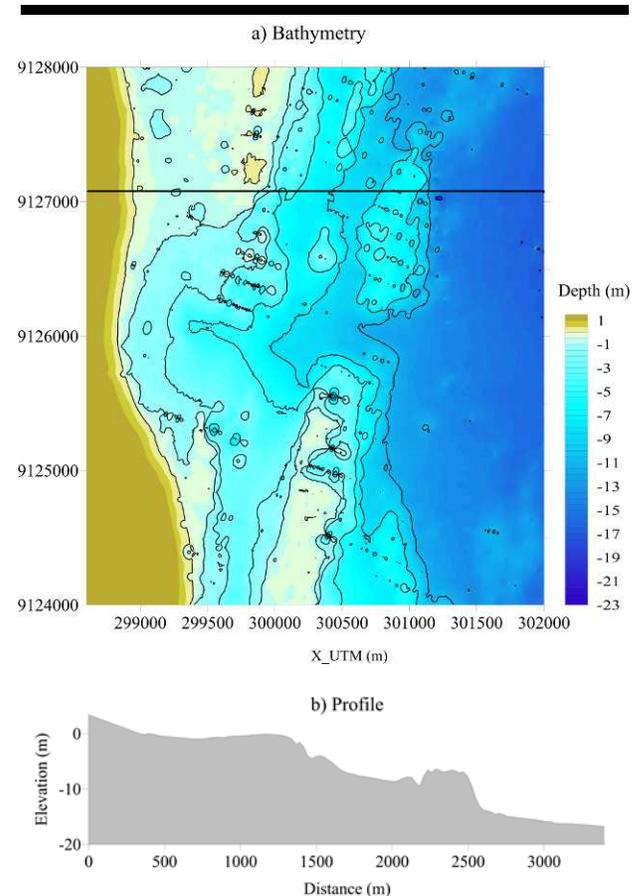


Figure 3. (a) Bathymetry in the study area and (b) cross shore profile, for the black line shown on the bathymetry.

Pianca *et al.*, (2010) described the Brazilian wave climate based on eleven-year time series hindcast data (Jan/1997-Dec/2007). In this study the authors concluded that the wave climate at Pernambuco coast is quite constant due to the low variation in wind velocity and direction (Figure 4).

Table 1 shows the wave probability obtained from a virtual wave buoy, located in front of Pau Amarelo beach, 2.5 km from the coast, at a depth of 8.97 m. This data is available from the Brazilian data bank provided by the Coastal Modeling System (Oliveira, *et al.*, 2013).

The predominant wave direction is from the East-Southeast

(ESE), which corresponds to the direction of the local wind. The significant wave height (H_s) is between 1.1 and 1.3 m and the corresponding period is between 5 and 7 s.

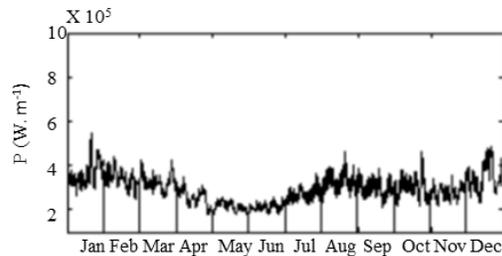


Figure 4. Daily averaged variations hindcast for the eleven-year time series of wave power (P) offshore of Pernambuco (Pianca *et al.*, 2010).

However this offshore data are not the same as that which reaches the coast. Several processes can modify the waves, such as refraction, which affects the wave direction. As well, part of the wave energy is dissipated over the reefs surface, then the wave height decreases. In addition, energy is also dissipated by bottom friction.

Table 1. Wave reanalysis data provided by SMCBrazil software (Oliveira *et al.*, 2013; <http://smcbrasil.ihcantabria.com/>)

Direction	Probability of Occurrence (%)	H_s (m)	T_p (s)
ENE	0.01	1.06	9.97
SE	1.76	1.16	5.93
E	7.36	1.16	7.57
ESE	90.86	1.29	7.19

The Southeast incoming waves determine the sediment transport by longshore drift to the North, along the coastline in the region.

Tide data is provided by the Brazilian Directory of Hydrography and Navigation, according to which the tidal range in the region is classified as mesotidal, with an average amplitude of 1.67 m. The mean amplitude during spring tide is 2.07 m and 0.97 m during neap tides, but it can reach 2.8 m during equinoctial tide. Due to its period, of 12.42 hours, the local tide is semidiurnal and results mainly from astronomical forcing, while the meteorological tide has no significant effect on the coast (FINEP/UFPE, 2009).

METHODS

Evaluation of Anthropogenic Interaction

To evaluate the anthropogenic impacts a report from the local community was considered, along with coastline variations from a Brazilian project called “Monitoramento Ambiental Integrado-MAI” (Integrated Environmental Monitoring) which was performed in 2009 (FINEP/UFPE, 2009).

The coastline position was determined using orthophotomaps from 1974 and 2008, with a scale of 1:2000. The shoreline was marked on the high water line, or on the limit between the wet and dry zones (FINEP/UFPE, 2009).

Wave Analyses

Two different numerical models were applied in order to reproduce the wave and wave driven currents: WAPO (Silva *et al.*, 2005) and COCO (Silva *et al.*, 2010), respectively. Both models have been successfully used in other studies.

The WAPO model is based on the modified mild-slope equation and can solve implicitly wave refraction, reflection, and diffraction. Therefore, this linear wave propagation model can solve physical processes in complex geometries (Mariño-Tapia *et al.*, 2011).

The COCO model solves the non-linear shallow water equations and uses the output of the WAPO model in order to calculate the radiation stresses and wave driven current fields.

The main statistical parameters, like wave height (H_s), peak period (T_p), wave direction (θ), and the mean tide level (MTL) were used as input for the models.

Since the study area has no variation in wave climate throughout the year (Table 1), only two scenarios were considered to simulate the wave propagation. One represents the standard condition, and other represents a more energetic situation, both scenarios are described below:

Scenario 1: $H_s = 1.3$ m, $T_p = 7$ s, $\theta = 112.5^\circ$ and MTL = 1.7 m.

Scenario 2: $H_s = 2.3$ m, $T_p = 9$ s, $\theta = 112.5^\circ$ and MTL = 2.8 m.

The data source for both scenarios is the SMC reanalysis. The first scenario is mean regime, while the second one uses the highest waves measured and its associated period, the direction of waves is considered the same in both scenarios (ESE) and the MTL is the highest measurement during equinoctial tide.

RESULTS

Possible Anthropogenic Causes of Erosion

Available literature for the area states that both natural and anthropogenic factors produce the local coastal erosion. A simplification of the four main factors was defined by Alveirinho Dias (1993) as follows:

- 1st. sea level rise,
- 2nd. decrease of sediment supply,
- 3rd. anthropogenic degradation of natural structures,
- 4th. hard engineering structures, particularly those implemented for coastal protection.

Several factors contribute to the erosion at Pau Amarelo. Although, the authors of this paper believe that the third factor of Alveirinho Dias (1993) is dominant, namely urbanization developing towards the sea. In the study area, practically all the existing dunes and backshore were destroyed for the construction of residential and commercial properties.

With the buildings very close to the sea, the backshore became sealed, and thus the second factor should be taken into consideration, as the natural sediment balance between land and sea was modified.

The fourth factor might also influence the erosion at this beach, since hard engineering structures were built in the south of Pau Amarelo. As the sediment transport is mainly from South to North, it is possible that these constructions diminish the sediment supply to the study area, however these implications require further investigations to confirm this. There are two main identifiable mechanisms that have modified the sediment supply:

In 1914 a jetty was built at Capibaribe River, the main source of continental sediment nearby, in order to increase the

navigability of the port located at the river mouth (see the yellow square in Figure 5). This jetty made the river plume disperse offshore, to an area farther away. Thus, the sediment might not stay in the coastal zone.

Immediately after the jetty was built, Olinda and Janga beaches (represented by white and blue squares, respectively, in Figure 5,) started to erode. To stop this erosion, several hard structures were built: 19 breakwaters and 35 groins. The construction of the groins immobilized the longshore drift, while the breakwaters promoted the formation of tombolos that also interrupted the longshore drift.

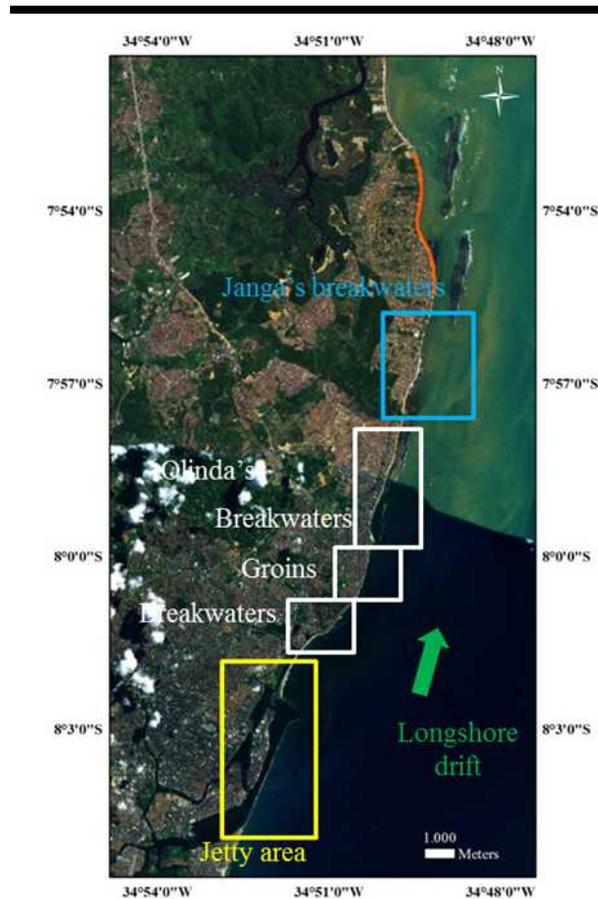


Figure 5. Location of hard engineering structures built south of Pau Amarelo Beach. The harbor area with the jetty is marked by the yellow box. The direction of the main longshore drift is shown by the green arrow. Two groups of breakwaters and a set of groins, at Olinda, are marked by a white box, and the breakwaters at Janga Beach are marked by a blue box. The present study coastline is highlighted in orange

Coastline Position

A coastline analysis performed in the framework of the MAI Project shows the shoreline displacement and the first line of houses between 1974 and 2008 (Figure 6a). A zoom at a portion with strong erosion shows, with more detail, the differences on the coastline position and occupation (Figure 6b and 6c). At

some points the coastline retreated by up to 20 m while in others accretion was registered.

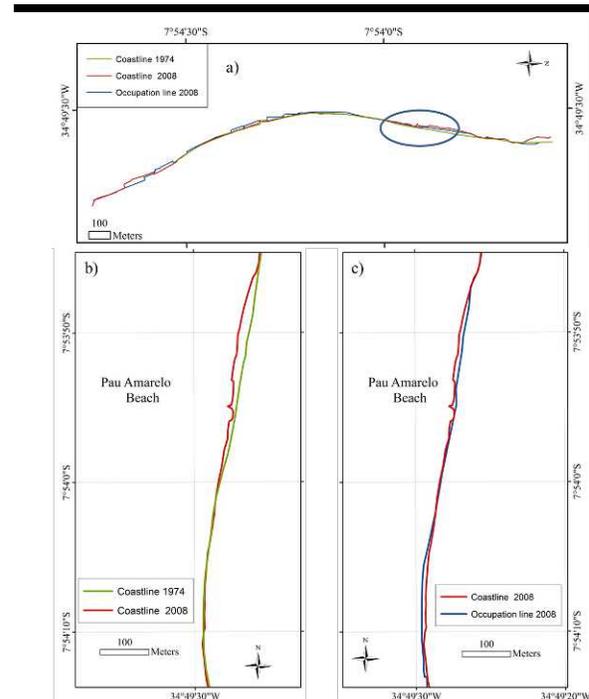


Figure 6. (a) Coastline position from 1974 and 2008, and occupation line from 2008 for the entire beach, (b) detail of the variation in coastline position in the north part of Pau Amarelo Beach and (c) detail of the relation between the coastline and the urban occupation line.

Wave Transformation

Besides the factors influencing the sediment supply already mentioned, wave refraction, reflection, shoaling, diffraction, and breaking, promoted by the reefs, might increase erosion by focusing wave energy in some places along the shore.

Figure 7 shows the results for the scenario most frequently found in Pau Amarelo beach, representing 90% of the cases, while Figure 8 shows a scenario of higher energy.

Figure 7a shows that in the beach central part the waves are the highest, whereas on the sides the waves are very low or nonexistent, because before reaching the beach, the energy is dissipated on the reefs.

In the second scenario (Figure 8) the waves can pass through the reefs and hit the entire coast of Pau Amarelo. However, the central part remains the region that receives the highest waves, and thus more energy. So this is the area of greatest susceptibility to erosion due to the higher transport capacity.

Figures 7b and 8b show the instantaneous free sea surface. In these it is possible to see the diffraction of waves caused by them passing through the reef faults. The refraction on the reefs, which causes an increase in wave height at these points (Figure 7a and 8a) also is visible. In accordance with the graphs of wave height, the free sea surface shows that in the second scenario the waves hit the entire coast, and not just the central part of the beach, as happens in the first case.

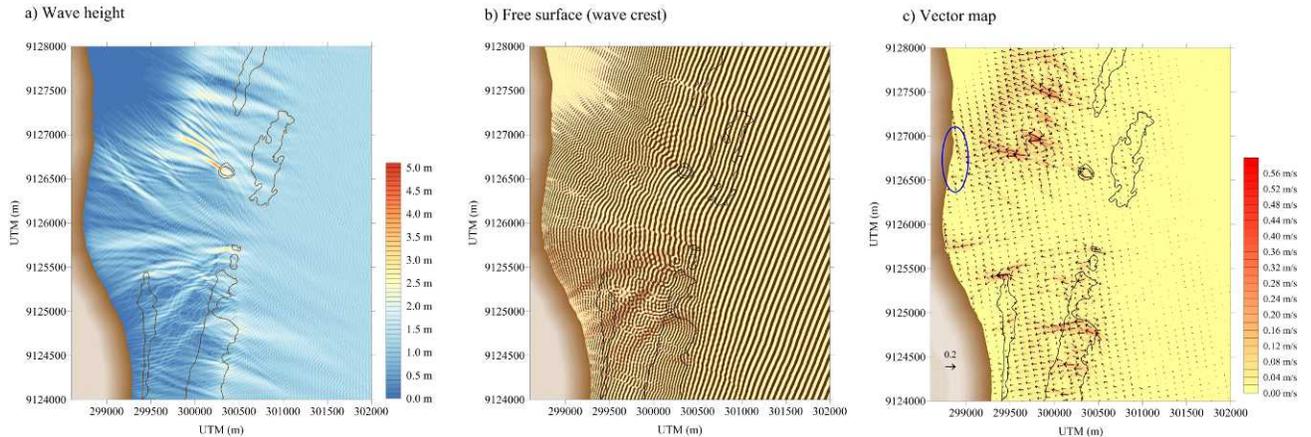


Figure 7. (a) Wave high pattern, (b) instantaneous free water surface and (c) computed wave-driven currents (for boundary conditions of $H_s = 1.3$ m, $T_p = 7$ s, $\theta = 112.5^\circ$, $MTL = 1.7$).

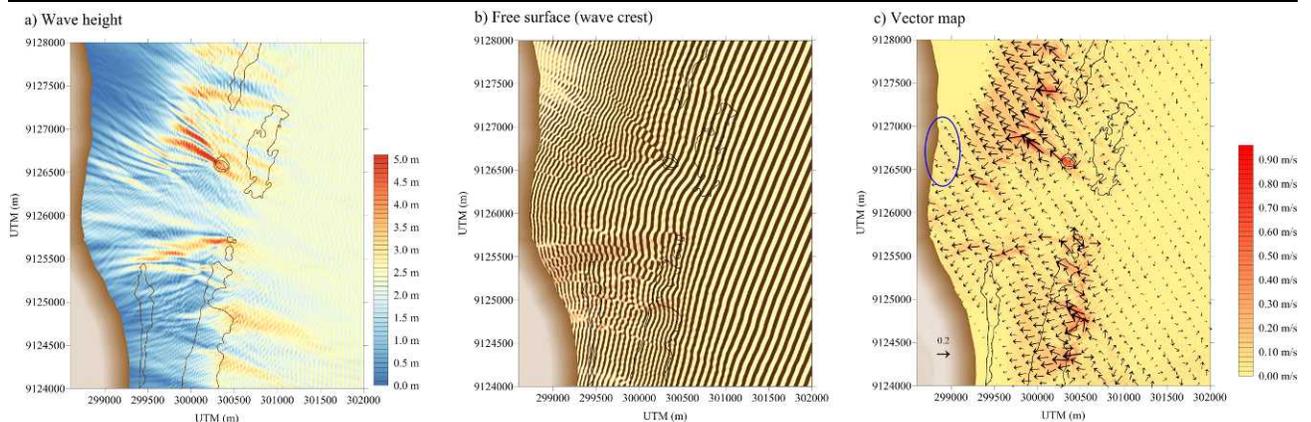


Figure 8. (a) Wave high pattern, (b) instantaneous free water surface and (c) computed wave-driven currents (for boundary conditions of $H_s = 2.3$ m, $T_p = 9$ s, $\theta = 112.5^\circ$, $MTL = 2.8$).

Figures 7c and 8c show the intensity and direction of the currents generated by waves. The currents in the second scenario are faster, but in both cases it is possible to see that the wave diffraction generates two main currents, diverging from the central region of the beach, one to north and other to south.

DISCUSSION

Since the construction of the hard structures, described earlier in this article, south of Pau Amarelo, the northern area may be receiving less sediment than before.

This is just a hypothesis and needs to be tested, but it is supported by local community reports, where they state that the erosion started to be feel like a “domino effect” after the construction of the jetty in Recife.

Besides the possible displacement of sediment supply, other alterations to coastal dynamics are caused by the construction of

buildings on the dune system. As a consequence, the sediment is confined by the urban development, thus altering the natural sediment balance of the beach. This is sometimes resulting in a negative budget, which means that the beach cannot restore its normal profile after periods of erosion.

Once the problem had begun, another aggravating factor was the construction of extensive seawalls along the coast. These structures increased the erosive process and the loss of beach sediments, leading to a reduction in beach levels. At high tide, waves reflect off the seawalls carrying sediments back into the ocean. During these periods, the increased capacity to remove sediments means that the seawalls can collapse.

Regarding the natural effects on the beach state, the wave behavior and the currents modeled in WAPO and COCO, respectively, can represent the dynamics of the near shore. Pau Amarelo Beach is affected by the processes of diffraction and

refraction. These govern the circulation and form longshore currents with the capacity to transport sediment.

According to the model results, the longshore current to the north is stronger than the one to the south, it is highlighted with a blue circle on the coast at Figures 7c and 8c. This area corresponds to the area of greatest erosion according to the shore line variation shown in Figure 6. The relation between both results corroborate to the reliability of the models output.

Finally, the circulation pattern, created by the wave refraction and diffraction processes at the reefs, determines the embayment shape of this beach, characterized by the growth of salients at the extremes of the study area being built from the sediment that comes from the central area.

Possible Solution: Beach Nourishment

There are many possibilities to protect a beach from erosion. It can be done with hard structures such as groins, seawalls and others, or with soft methodologies, such as sand by-passing or artificial nourishment.

Considering the maintenance of the natural beauty of the beach, soft methods such as beach nourishment or sand by passing are often preferable rather than hard engineering solutions.

Beach nourishment is the most common soft methodology implemented in many countries, offering protection and maintaining the viability of the shorelines for anthropogenic and ecological services, as well as restoring the beach, and protecting the environment behind it (Anthony *et al.*, 2011).

This technique involves adding sand to increase the width of the eroded beach, reestablishing the width to what it was before erosion. At Pau Amarelo the natural width is considered to be the coastline of 1974, shown in Figure 6.

The sediment for the nourishment can be provided from marine deposits or even from continental ones; the important fact is that this source needs to be close to the beach, otherwise the work becomes too expensive.

As a first approach, to calculate the volume of sediment necessary to restore the beach, the methodology proposed by Campbell *et al.* (1990) can be used, by solving the equation below:

$$V_T = (B + H_c)L \cdot W \quad (1)$$

where V_T is the total volume necessary to nourish the beach [m^3/m], B is the berm elevation [m], H_c is the closure depth [m], L is the longshore length of the nourishment project [m], and W is the desired amount of beach widening.

Since beach profiles at Pau Amarelo are available from the MAI Project, B is set as 3 m. Figure 6b shows a coastline retreat from 1974 of up to 20 m. Which gives a W value of 20 m.

The closure depth was calculated according to Dean (1977):

$$H_c = 2.28H_e - 68.5 \left(\frac{H_e^2}{gT_e^2} \right) \quad (2)$$

where H_c represents the closure depth [m], H_e the effective wave height [m], g is the acceleration due to gravity [m/s^2], and T_e is the effective period [s].

Near shore wave characteristics (*i.e.* after passing the reefs) are required to calculate H_e . As the only wave data available are from beyond the reefs, it is necessary to use the values from wave model propagation. For this purpose the input to the WAPO model was used, the wave parameters from SMC-reanalysis, as follows: $H_{s12} = 2.6$ m, $T_{p12} = 14.257$ s, $\theta = 112.5^\circ$ and $MTL = 2.8$ m.

With these results it was possible to determine H_e for the area: 0.90 m, while T_e is 14.25 s, due to the conservative properties of waves T_e remains the same as beyond the surf zone.

Solving equation (2), the estimated closure depth is 2.24 m.

The longshore length can be obtained from the bathymetry, by measuring a line from the coast to the closure depth. Thus L is 515 m from the coast.

Taking that into account, the cross shore length is 4 km, the estimated total volume of sand required is 207,000.00 m^3 .

This sediment can be added to the beach face and, after some time, the previous beach profile will be regained naturally by wave action. Particular attention needs to be given to the particle size of the sand used, sorting and composition, since the density of the sediment has to be equal or larger than that of the original beach sediment. If the sediment to be used is less dense than the original, it will easily be eroded (CBNP, 1995).

Although beach nourishment is an alternative with low environmental impact, it provides only a temporary solution to the erosion. The frequency of maintenance nourishment is related to the beach dynamics.

This evaluation is a first approach, giving only a general idea. Before implementing a nourishment project it is necessary to model all the morphodynamic conditions of the area.

CONCLUSIONS

The analysis of the wave propagation pattern performed by means of the WAPO and COCO models contributed significantly to improve the understanding of the local dynamics, since no measurements of this parameters are available for the region between the beach and the reefs at Pau Amarelo.

Pau Amarelo is a heavily urbanized beach, with a completely changed of the coastal zone, which makes it difficult to identify the real causes of erosion-whether they are natural, anthropogenic, or a combination of both.

However, the analysis performed indicates, that the occupation of the backshore and the existence of protection structures on the coast are the main factors driving the erosional processes along most of the beach.

At some points of the coast, the erosion is intensified by circulation pattern, and anthropogenic pressure. This is caused by the unbalance between the sediment transport produced by the longshore currents and sediment deficit, generated by the protection structures.

The Pau Amarelo study shows a case where erosion is less intense where the waves are higher. As the location that has strong erosion has low waves, on the other hand a stronger current.

Beach nourishment is considered by the authors of this paper as the best solution for the area, since it works with the nature and not against it.

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